(7 pages) **Reg. No. :** ....

Code No. : 5853 **Sub. Code : PMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2020.

Fourth Semester

Mathematics – Core

COMPLEX ANALYSIS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The radius of convergence of the series $\sum \lfloor\!\underline{n}\, Z^n$ is

   (a) 0

   (b) ∞

   (c) e

   (d) 1

2. If $f(z) = u + iv = |z|^2$ then $\dfrac{\partial u}{\partial y} =$ ________

(a) $2x$ (b) $2y$

(c) $-2y$ (d) $-2x$

3. The transformation $\dfrac{1}{z}$ is called ________.

(a) parallel translation

(b) inversion

(c) rotation

(d) homothetic transformation

4. Which one of the following is false?

(a) two reflections result is a linear transformation

(b) reflections are linear transformations

(c) $w = z + \alpha$ is called a parallel translation

(d) $\operatorname{Im}(Z) = -\operatorname{Im}\left(\dfrac{1}{Z}\right)$

5. $\displaystyle\int_{|z|=1} \frac{e^z dz}{z} =$

(a) 1 (b) 0

(c) $2\pi i$ (d) none of these

6. If C is the circle $|z-2|=5$, then $\int_C \frac{dz}{z-3}$ is

(a) $2\pi i$ (b) 0

(c) 1 (d) 2π

7. $f(z)=\frac{1}{z}$ has a removable singularity at $Z=$ ______________.

(a) 1 (b) ∞

(c) 0 (d) none

8. For the function $f(z)=\frac{1-e^{2z}}{z^4}$, the point $Z=0$ is a pole of order.

(a) 4 (b) 3

(c) 1 (d) ∞

9. $\int_0^{\pi} \log \sin x \, dx =$ ______________

(a) $\pi \log 2$ (b) $-2\pi \log 3$

(c) $-\pi \log 2$ (d) none of these

10. The number of roots of the equation $z^7 - 2z^5 + 6z^3 - z + 1 = 0$ in the disc $|z| < 1$ is

(a) 3 (b) 4

(c) 5 (d) 7

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the necessary condition for differentiability.

Or

(b) State and prove Lucas theorem.

12. (a) At each point z for a region Ω where $\mathcal{C}(z)$ is analytic and $f'(z) \neq 0$. Then prove that the mapping $w = f(z)$ is conformal.

Or

(b) Find the linear transformation which carries 0, i, $-i$ into 1, -1, 0.

13. (a) Compute $\int\limits_{|z|=2} \frac{dz}{z^2+1}$ by decomposition of integral into partial fraction.

Or

(b) State and prove Cauchy's Integral formula.

14. (a) State and prove fundamental theorem of algebra.

Or

(b) State and prove Weierstrass theorem for essential singularity.

15. (a) State and prove Rouche's theorem.

Or

(b) State and prove the argument principle.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that every rational function has a representation by partial fraction.

Or

(b) Prove that a rational function $R(z)$ of order p has p zeros and p poles and also prove that every equation $R(z) = a$ has exactly p roots.

17. (a) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line. Also prove that $z \to \bar{z}$ is not a linear transformation.

Or

(b) Prove that the integral $\int_{\gamma} f(z)\,dz$, with continuous *f*, depends only on the end points of γ if and only if $\mathfrak{C}$ is the derivative of an analytical function in Ω.

18. (a) State and prove Cauchy's theorem for a disk.

Or

(b) Define winding number and write three properties of winding number.

19. (a) Prove that an analytic function has derivatives of all orders.

Or

(b) Prove that a non constant analytic function maps open sets onto open sets.

20. (a) Evaluate $\int_0^\infty \frac{\log(1+x^2)\,dx}{x^{H\alpha}}$, $0<\alpha<2$.

Or

(b) Prove that $\int_0^\infty \frac{\sin mx}{x}dx = \frac{\pi}{2}$.